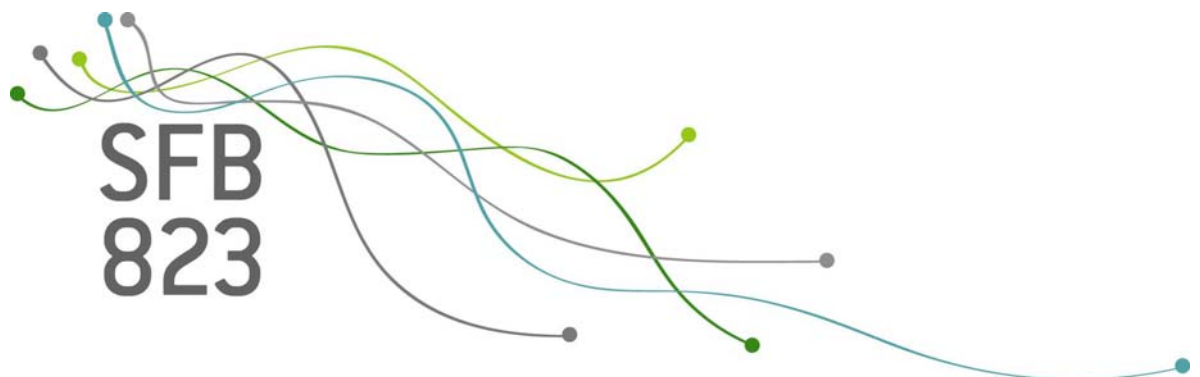


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Structural change and spurious persistence in stochastic volatility

Walter Krämer, Philip Messow

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Discussion Paper

Structural Change and Spurious Persistence in Stochastic Volatility ¹

by

Walter Krämer

Fakultät Statistik, Technische Universität Dortmund, Germany

Phone xx231/755-3125, Fax: xx231/755-5284

e-mail: walterk@statistik.tu-dortmund.de

and

Philip Messow

Ruhr Graduate School in Economics

RWI, Hohenzollernstr. 1-3, 45128 Essen

Phone xx231/755-5246, Fax: xx231/755-5284

e-mail: messow@statistik.tu-dortmund.de

Abstract

We extend the well established link between structural change and estimated persistence from GARCH to stochastic volatility (SV) models. Whenever structural changes in some model parameters increase the empirical autocorrelations of the squares of the underlying time series, the persistence in volatility implied by the estimated model parameters follows suit. This explains why stochastic volatility often appears to be more persistent when estimated from a larger sample as then the likelihood increases that there might have been some structural change in between.

JEL classification: C32, C58

Keywords: Persistence, Stochastic Volatility, Structural Change

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1 Introduction

In the context of the GARCH-family of discrete time series models, it is a well established empirical fact that the persistence of the volatility tends to increase with the length of the sample – in calendar time – that is used for the estimation of the model parameters (Lamoureux and Lastrapes (1990), Krämer and Tameze (2007)). According to Diebold (1986), this upward tendency is often due to a switch in regimes somewhere in the sample, and the probability of a switch increases with increasing calendar time. Krämer and Tameze (2007), Mikosch and Starica (2005), Hillebrand (2005), Krämer (2008) and Krämer et al. (2011) explore the mechanics of the relationship for various stochastic and nonstochastic types of structural change.

The present paper considers the following simple stochastic volatility (SV) model and shows that similar mechanisms are at work here as well:

$$y_t = \sqrt{h_t} \xi_t + \mu, \quad (t = 1, \dots, T) \quad (1.1)$$

$$\log h_t = \phi + \delta \log h_{t-1} + \sigma \varepsilon_t, \quad (1.2)$$

where $\mu = E(y_t)$, $|\delta| < 1$ and ξ_t and ε_t are *iid* $N(0, I_2)$. This model is also known as the ARSV(1)-model. Our results extend in a straightforward manner to more complicated SV models. In the context of the simple model above, it can be shown (see e.g. Carnero et al. (2004)) that the rate of decay of the autocorrelations of $(y_t - \mu)^2$ tends to δ as time lags are increasing, so this parameter is a measure of the persistence of shocks to volatility in the model described by (1.1) - (1.2).

Similar to the GARCH class of models, the estimated persistence tends to increase with the length of the sample in calendar time also among SV-models: When fitted to empirical data, the estimator $\hat{\delta}$ of the persistence parameter

δ is close to, but less than 1 and increases with increasing sample size. The two most common applications are exchange rates and stock returns. Carnero et al. (2004) obtain persistence parameters of 0.8499 (T=1,262) to 0.9781 (T=2,888), Taylor (1994) estimates a persistence of 0.9719 (T=3,283). Andersson (2001) obtains value ranging from 0.9577 (T=2,134) to 0.9870 (T=2,173), while Shepard's (1996) estimates range from 0.936 (T=2,113) to 0.967 (T=2,160)(see Figure 1).

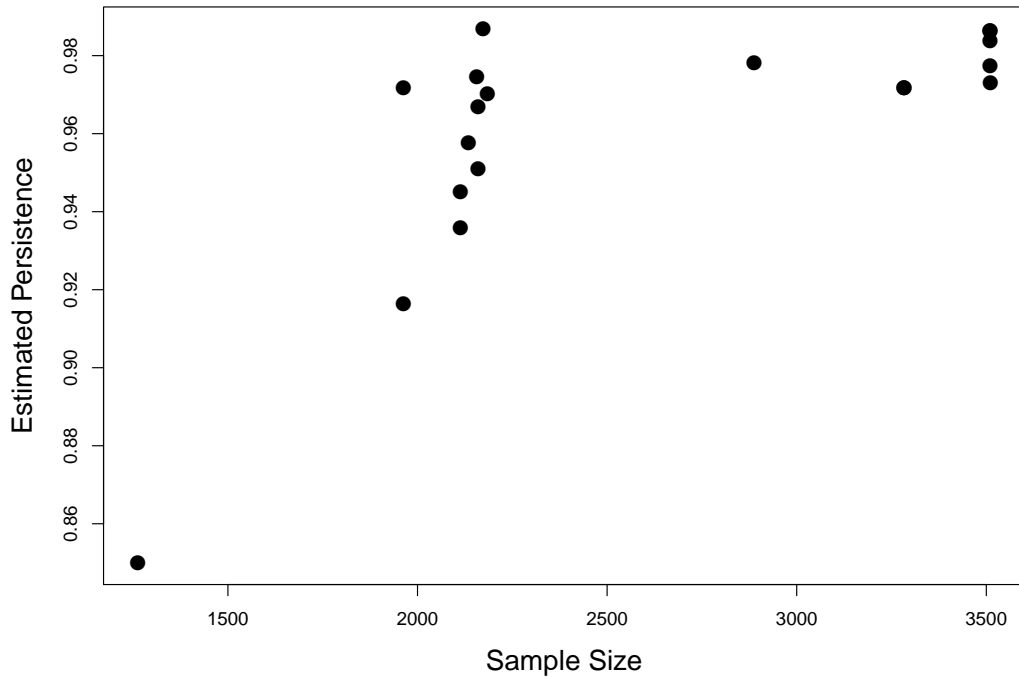


Figure 1: Estimated persistence and sample size in the ARSV(1)-model

Psaradakis and Tzavalis (1999) already observed that such increases in estimated persistence might be caused by structural changes in the model param-

eters, no matter which estimator for δ is used. Below we consider the following closed-form estimator for δ :

$$\hat{\delta}_T = \frac{\hat{\rho}_{2,T}}{\hat{\rho}_{1,T}}, \quad (1.3)$$

where $\hat{\rho}_{1,T}$ and $\hat{\rho}_{2,T}$ are estimators for the first and second order autocorrelations of $z_t := \log(y_t^2)$ from a sample of size T . It can be shown (see for instance Hafner and Preminger (2010), Theorem 1) that $\hat{\delta}_T$ is consistent and asymptotically normal when the data generating process is as described in (1.1) and (1.2).

Here we are interested in the behavior of $\hat{\delta}_T$ when there is a change in the values of ϕ , δ , μ or σ somewhere in the sample. Extending Psaradakis and Tzavalis (1999), we show that $\hat{\delta}_T$ can be made arbitrarily close to 1 if either the sample or the structural change is large enough. As $\hat{\delta}_T$ is basically the ratio of the second to the first order empirical autocorrelation coefficient of the logs of the squared observations, any change that will make these empirical autocorrelations equal to each other will induce an increase in the estimated persistence, as then $\hat{\delta}_T \xrightarrow{P} 1$. Section 2 points to various ways in which this can happen, section 3 illustrates the magnitude of such effects via some selected Monte Carlo experiments and section 4 considers some extensions.

2 Structural change and empirical autocorrelation of the logs

From formula (1.3) above, it is evident that the estimated persistence, at least for the estimator we consider here, is a function of the empirical autocorrelations of $z_t = \log(y_t^2)$. Now it is well known (see e.g. Hassler (1997)) that the empirical autocorrelations of z_t

tend to one in probability whenever z_t exhibits nonstationary long memory. To the extent therefore that (seemingly) nonstationary long memory in $\log(y_t^2)$ is induced by structural changes, the estimator $\hat{\delta}_T$ from (1.3) will likewise tend to one. Krämer et al. (2011) discuss various ways in which such (seeming) nonstationary long memory can be produced.

For any given sample size T , Krämer and Tameze (2007) show that empirical autocorrelations of y_t^2 will also tend to one in probability when $\mu \rightarrow \mu + \Delta$ at some fraction of the sample as Δ increases and it is easily seen that the same applies to $z_t := \log(y_t^2)$. However, structural changes of that magnitude appear unlikely in practice. More generally, consider the sample autocorrelation function in a situation where $r - 1$ structural breaks in any of the parameters ϕ , δ , σ or μ occur at $[Tq_1]$, $[Tq_2]$, ..., $[Tq_{r-1}]$, $q_0 := 0 < q_1 < q_2 < \dots < q_{r-1} < 1 =: q_r$. The only condition is that this change must affect $E(z_t)$. There are then r regimes, of duration Tp_j each, where $p_j = q_j - q_{j-1}$ ($j = 1, \dots, r$). Let $E^{(j)}$ be the expectation of z_t and $\gamma_k^{(j)}$ be the k -th order autocovariance of z_t in regime j (assuming that second moments of z_t exist in each regime). From Mikosch and Starica (2004, formulae 5), it is obvious that then

$$\hat{\delta}_T \xrightarrow{p} \frac{\sum_{j=1}^r p_j \gamma_2^{(j)} + \sum_{1 \leq i < j \leq r} p_i p_j (E^{(j)} - E^{(i)})^2}{\sum_{j=1}^r p_j \gamma_1^{(j)} + \sum_{1 \leq i < j \leq r} p_i p_j (E^{(j)} - E^{(i)})^2} \quad (2.4)$$

as $T \rightarrow \infty$. However, both the numerator and the denominator of this ratio are dominated by the respective second term when structural changes become large, so the ratio must then tend to 1.

3 Some Monte Carlo Simulations

Next we check the finite sample relevance of the above result by some Monte Carlo experiments. Table 1 reports the expected value of $\hat{\delta}_T$ (from (1.3)) as

Table 1: Impact of a structural break in μ on estimated persistence ($T = 5000$)

q	$\Delta\mu$					
	0	0.5	1	2	5	10
0.25	0.667	0.829	0.898	0.960	0.990	0.994
0.5	0.667	0.712	0.835	0.934	0.986	0.992
0.75	0.667	0.716	0.785	0.892	0.973	0.985

Table 2: Impact of a structural break in ϕ on estimated persistence ($T = 5000$)

q	$\Delta\phi$					
	0	1	2	3	4	5
0.25	0.667	0.933	0.980	0.990	0.994	0.996
0.5	0.667	0.951	0.986	0.993	0.996	0.997
0.75	0.667	0.932	0.979	0.990	0.994	0.996

obtained from 1000 Monte Carlo runs, for $\phi = 0.3$, $\delta = 0.6$, $\sigma = 0.5$, $\mu = 0$ and a single structural break in μ at $t = 1250$, $t = 2500$ and $t = 3750$ respectively. It is seen that δ is estimated almost unbiasedly when there is no structural change, but that the estimator tends to 1 as the structural change increases, no matter where the change occurs.

Table 2 gives the analogous results for a structural change in ϕ . Results are even more pronounced here, as a change in ϕ translates exponentially into a change in $E(z_t)$, so a remarkable upward bias is obvious here as well. Similar results (available upon request), were also obtained for other parameter combinations and other sample sizes T .

4 Discussion

Our theoretical argument relies crucially on the particular form (1.3) of the estimator of the persistence parameter δ . There are various competitors where the mechanics which drive a potential upward bias are not as clear. We did some Monte Carlo experiments for these estimators as well and found that they are likewise tending to one in the context of the structural changes we consider here.

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